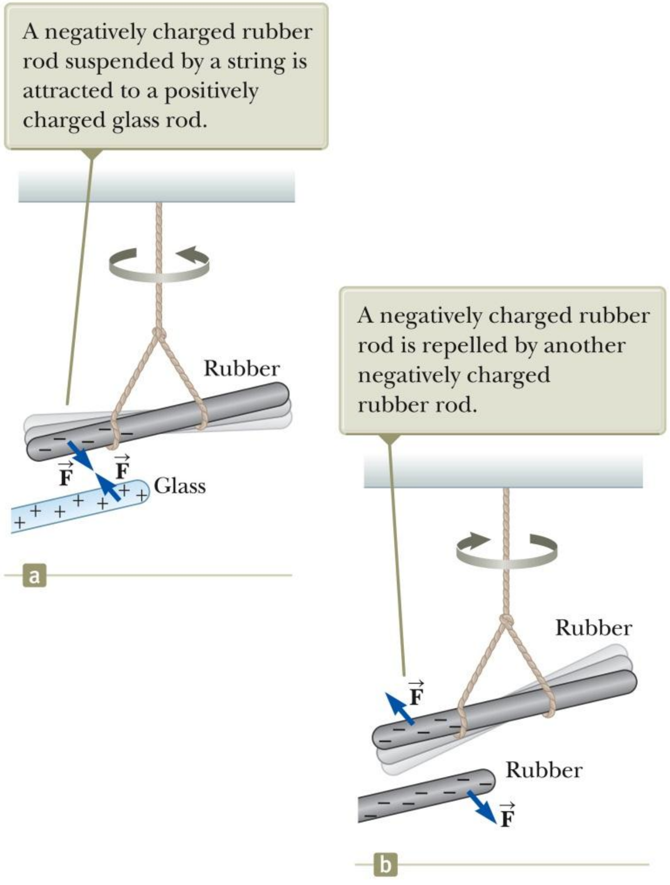
Overall Story

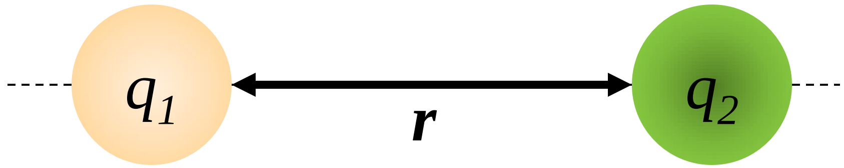
- Protons and electron carry charge
- Presence of charge creates an electric field
- The field creates potential differences between points within the space it exists
- Other charges react to this field and potential and move about (current)

Electrical Charges

- Charge is carried by electrons and protons.
- Can be positive or negative.
- Like charges repel, opposite charges attract.
- Total charge in a system is conserved.
- Charges come in discrete quantities.
- Charges are measured in Coulombs (C).
- Usually denoted by q.



Electric Force and Field



$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ \mathrm{N.m^2/C^2}$$

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_{\mathbf{e}}}{q_0} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/N.m^2}$$

E-Field

$$\vec{\mathbf{E}}$$

→

Force

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

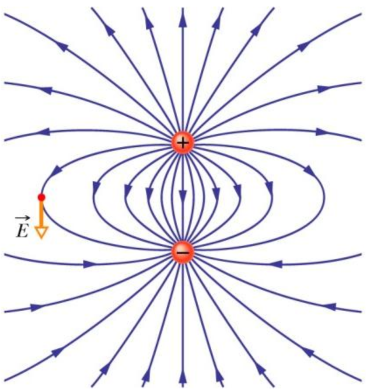
→

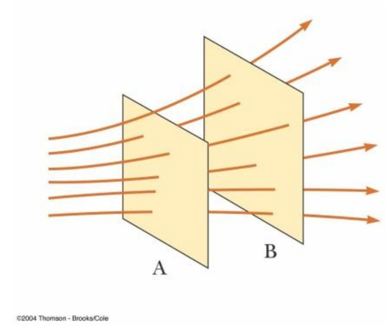
Acceleration

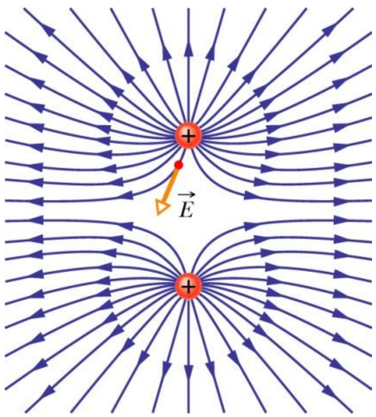
$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m}$$

Electric Field Lines

- A way to visualize field patterns over space.
- The e-field is tangent to the field lines at each point and along the direction of the field arrow.
- The density of the lines is proportional to the magnitude of the e-field.
- Field lines start from positive charges an end at negative ones.
- Field lines can not cross.







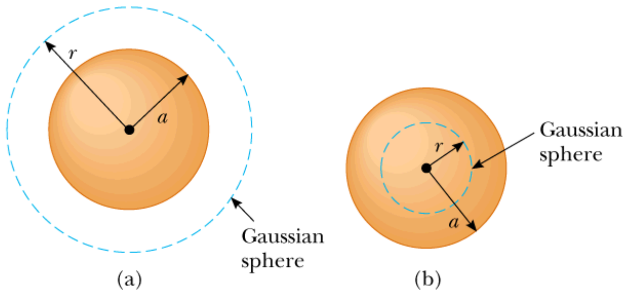
Gauss' Law

Electric Flux

$$\boxed{\Phi_E = \int_{surface} \vec{\mathbf{E}}.d\vec{\mathbf{A}}}$$

Gauss' Law

$$\boxed{\Phi_E = \oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}}$$

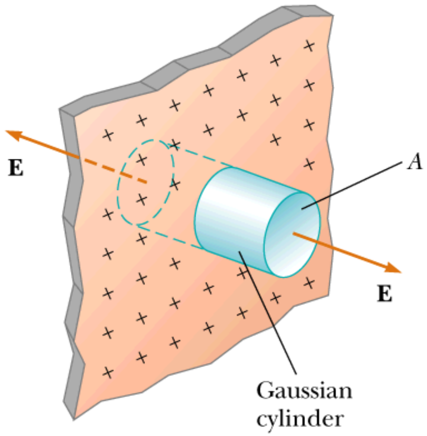


$$E = k_e \frac{Q}{r^2}$$

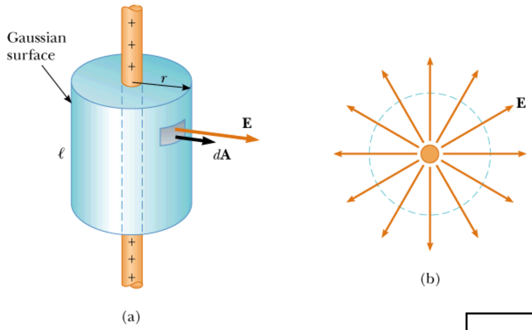
r > a

$$E = k_e \left(\frac{Q}{a^3}\right) r$$

r < R



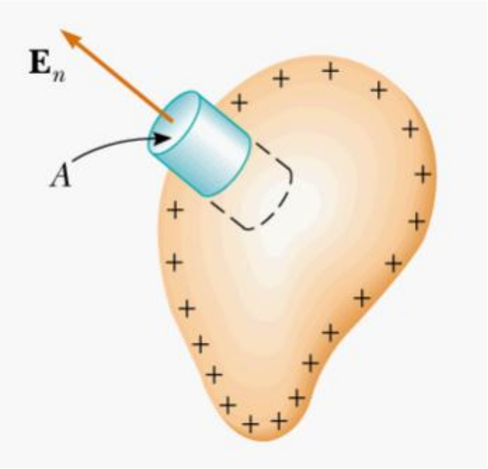
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$



$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Conductors and Gauss' Law

- The electric field is zero everywhere inside a conductor at electrostatic equilibrium.
- Any net charge on a conductor will reside on the surface.
- The electric field just outside a conductor is perpendicular to the surface and is proportional to the charge density.
- The charge density is highest near parts of the conductor with the smallest radius of curvature.



$$E = \frac{\sigma}{\varepsilon_0}$$

Electric Potential

Electric Potential Energy $$\boxed{\Delta U = -q_0 \int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}}$$

Potential Difference $$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

In a Uniform Field $$\Delta V = -Ed$$

For a Point Charge $$\boxed{V = k_e \frac{q}{r}}$$

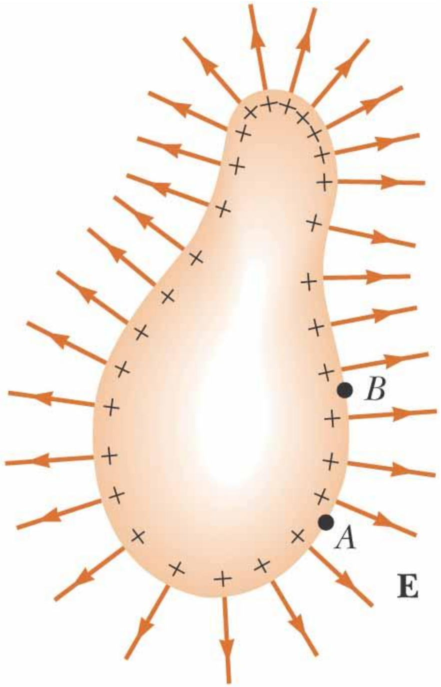
For a Charge Distribution $$\boxed{V = k_e \int \frac{dq}{r}}$$

$$E_x = \frac{\partial V}{\partial x}$$

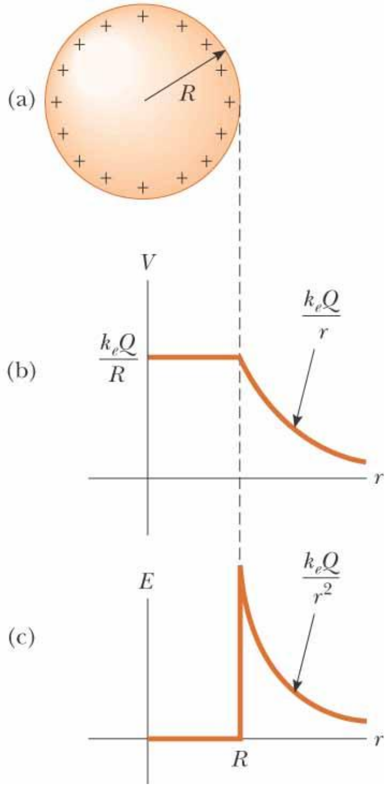
$$E_y = \frac{\partial V}{\partial y}$$

$$E_z = \frac{\partial V}{\partial z}$$

Potential Due to a Charged Conductor



- Charges always reside at the outer surface of the conductor.
- The field lines are always perpendicular to surface.
- Then **E**.*d***s**=0 on the surface at any point.
- Which means, V_B-V_A=0 along the surface.
- The surface is an equipotential surface.
- Finally, since **E**=0 inside the conductor, the potential *V* is constant and equal to the surface value.



Capacitors

$$\boxed{C \equiv \frac{Q}{\Delta V}}$$

(C/V=F)

Parallel Plate Capacitor

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}}$$

For series capacitors

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$

For parallel capacitors

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j}$$

Stored Energy in a Capacitor

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2}$$

Capacitor with a dielectric

$$\boxed{C = \kappa C_0}$$

Current and Resistance

$$I(t) = \frac{dQ}{dt}$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

$$R = \frac{\Delta V}{I}$$

$$\rho = \frac{1}{\sigma} \quad R = \rho \frac{l}{A}$$

$$R = R_0 \left[1 + \alpha \left(T - T_0 \right) \right]$$

$$\mathscr{P} = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

DC Circuits

Series

$$R_{eq} = \sum_i R_i$$

Parallel

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Kirchoff's Rules

$$\sum_{junction} I_{in} = \sum_{junction} I_{out}$$

$$\sum_{\substack{closed\\loop}} \Delta V = 0$$

RC Circuits

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q\left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$q(t) = Qe^{-t/RC}$$

$$I(t) = \frac{dq}{dt} = -\frac{Q}{RC} e^{-t/RC}$$